Explaining the energie-efficiency paradox: a vintage model with returns to diversity and learning-by-using

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Explaining the Energy-Efficiency Paradox.
A Vintage Model with Returns to Diversity and Learning-by-Using

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Abstract

This paper studies the adoption and diffusion of energy-saving technologies in a vintage model. An important characteristic of the model is that vintages are modeled as being complementary: there are returns to diversity of using different vintages. We analyse how diffusion patterns and adoption behaviour are affected by complementarity and learning-by-using. It is shown that the stronger the complementarity between different vintages and the stronger the learning-by-using, the longer it takes before firms scrap (seemingly) inferior technologies. We argue that this is a potentially relevant part of the explanation of the energy-efficiency paradox. Furthermore we explore the effects of energy tax policies.

1. Introduction

Concerns about global climate change associated with the combustion of fossil fuels urge a call for the development and widespread adoption of energy-saving technologies. It is beyond doubt that the *development* of new energy saving technologies - often labelled with the subsequent phases of invention and innovation - plays an important role in meeting policy targets with respect to the stabilisation or reduction of greenhouse gas emissions. The *diffusion* of existing technologies, however, is at least equally important, costly and difficult, as the development of new technologies (see, for example, Jovanovic, 1997). It has indeed been shown that the widespread adoption of existing energy-saving technologies could enable a significant reduction in energy use, especially in the short and medium term (see for example de Beer 1998, IWG 1997). At the same time, however, it is known that diffusion of new technologies is a lengthy process, that adoption of new technologies is costly and that many firms continue to invest in old and (seemingly) inferior technologies. The latter phenomenon has become known as the energy-efficiency paradox: the existing gap between the most energy-efficient technologies available at some point in time and those that are actually in use (Jaffe and Stavins 1994, Jaffe et al. 1999). The aim of this paper is to contribute to our understanding of adoption behaviour of firms and of diffusion processes of new energy-saving technologies in order to improve our understanding of the energy-efficiency paradox.

The question why firms do not invest in seemingly superior technologies has already achieved much attention in the literature. We can distinguish four major explanations for the relatively slow diffusion of new technologies. The first explanation is that the combination of uncertainty and some degree of irreversibility in investment creates an option-value of waiting (see, for example, Balcer and Lippman 1984, Dixit and Pindyck 1994 and Farzin et al. 1998). The second explanation deals with strategic issues: in a world characterised by spill-overs and limited appropriability, the presence of (expected) rival innovation and imitation creates an argument for firms to postpone innovation or adoption (see, for example, Kamien and Schwartz 1972, Reinganum 1981). The third explanation emphasises the fact that over time the performance of existing technologies improves and their price reduces due to learning and spillover effects (see, for example, Jovanovic and Lach 1989 and OECD/IEA 2000). A final explanation emphasises the role of vested interests. As switching to new technologies (temporarily) reduces expertise and destroys rents associated with working with relatively old technologies for particular subgroups in the economy, these groups may engage in efforts aimed at keeping the old technologies in place (see, for example, Canton et al. 1999, Jovanovic and Nyarko 1994, Krusell and Ríos-Rull 1996).

We offer two explanations for the slow diffusion of energy-saving technologies that complement the before-mentioned explanations. This is done by developing a simple two-sector macro-economic model including a final goods sector, producing a homogenous consumption good, and a capital production sector producing heterogeneous vintages. The first explanation is rooted in a complementarity effect and the second one in a learning effect with the user of the technology. The distinctive features of our model are as follows[1]. First, technology is embodied in physical capital. Newer vintages need less labour and energy to produce the same output which leads to a vintage structure of production. Second, capital of different vintages are imperfect substitutes in production. The economy exhibits a 'taste for diversity' of vintages creating an incentive to invest in both new and older technologies. Third, the representative firm in the final goods sector gains expertise in a technology by using the technologies in its production process. In other words, we include learning-by-using.[2] Fourth, our model allows for the endogenous determination of the number of vintages used in the final goods sector, so we offer an economically motivated approach for the scrapping of vintages.

In contrast with traditional vintage models, our model exhibits a 'taste for diversity' of vintages. We argue that complementarity is not so much a by-product of past investment decisions, but an essential ingredient in the process of technological change. Many new technologies pass through a life cycle, in which they initially complement older technologies, and only subsequently (and often slowly) substitute for the older technologies. A number of historical examples, like for example the replacement of the waterwheel by the steam engine, illustrates the role of complementarities in this 'life cycle view' of technological change (see for example Rosenberg 1982, Young 1993b).

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1 The model presented in this paper essentially extends the basic framework developed in De Groot, Hofkes and Mulder (2000). The extension consists of incorporating energy as a production factor into the model.

2 We follow Rosenberg (1982:121-122) in distinguishing three basic types of learning: learning in R&D stages, learning at the manufacturing stage and learning as a result of use of the product. We refer to the second type as learning-by-doing and to the last type as learning-by-using. In our model, the final goods sector gains experience in using capital goods.

One can argue that modern production processes consist of even more interrelated and mutually reinforcing technologies than the documented historical examples. Whereas Young (1993b) employs the idea of complementary innovation, we focus on the complementarity effect in diffusion processes. It is evident that at the economy level there is continuous investment in both old and newly arrived technologies. It can be argued that this pattern also exists at the sector or even the firm level, depending on the technology and the type of production process. It is this complementarity effect that is at the heart of our model.

The taste for technological diversity may further be intensified by the learning-by-using effect in the final goods sector. In accordance with broad historical evidence (see for example Mokyr 1990, Rosenberg 1982 and Young 1993a) we allow for new technologies to be inferior initially to more mature technologies. Learning-by-using improves the productivity of the new technology over time. A switch of technologies temporarily reduces expertise and counter effects the improved potential productivity level and the decreased energy-capital ratio of newer vintages. The prospect of such a productivity drop prevents agents from immediate and 'total' switching, but rather induces a gradual adoption of new technologies resulting in coexistence of old and new technologies.

There are a number of related articles in which issues of learning and technological innovation and diffusion are analysed. Without extensive discussion, we refer to, for example, Aghion and Howitt (1996), Aghion et al. (1997, 1999), Arrow (1962), Chari and Hopenhayn (1991), Jovanovic and Nyarko (1996), Parente (1994), Stokey (1988) and Young (1993a,b). The main differences between these articles and ours lie in the specifications we make with respect to the complementarity of vintages, the interpretation of intermediate goods as technologies, the emphasis on diffusion instead of innovation, the endogenous scrapping mechanism and, finally, the inclusion of energy-biased technological change. In focussing on diffusion and adoption our paper differs from most literature on induced or endogenous technological change in which the focus is almost solely on innovation (see, for example, Goulder and Schneider 1999 and Goulder and Mathai 2000).

The paper is organised as follows. In section 2 we set up the basic model. In section 3 we derive the solution of the model. Section 4 contains some comparative statics including an analysis of the effects of learning and taxation. Section 5 concludes.

2. The model

The model that we develop is essentially a simple two-sector vintage model that is characterised by learning-by-using and 'returns to diversity'. The two sectors that we distinguish are (i) a final goods sector in which a homogeneous consumption good is produced using labour, capital and energy, and (ii) a capital goods sector consisting of *T* monopolistically competitive firms each producing a unique vintage of capital. Labour is used for assemblage of final consumption goods and the production of capital or intermediate inputs. Energy is a complement to capital. For simplicity, capital is assumed to be non-durable. The model can hence also be considered as a model with heterogeneous intermediate inputs. These intermediates or vintages are assumed to be imperfect substitutes. The formulation that we use in our model to capture this is inspired by the product-variety theory which started with the seminal work of Dixit and Stiglitz (1977) and was later extended and applied by, for example, Ethier (1982), Grossman and Helpman (1991) and Romer (1990). An advantage - at least for presentational purposes - of this assumption is that the coexistence of different vintages can, by definition, not be explained on the basis of incomplete depreciation of the existing capital stock as is common in 'traditional' vintage models. Furthermore, our vintage approach offers an attractive framework to analyse energy-efficient technology diffusion because it allows us to include investment decisions at the firm level and (energy) characteristics at the technology level in a macro-economic setting.

2.1 The final goods production sector

The final goods sector produces a homogeneous consumption good according to a Cobb-Douglas production function:

$$Y_t = K_t^{\alpha} L_{Yt}^{1-\alpha} \tag{1}$$

in which Y_t represents output produced in year *t*, and K_t and L_{Yt} are the capital- and labour input in final goods production, respectively. Energy is assumed to be complementary to capital (we return to this below). Capital is an aggregate of vintages of capital goods. Vintages are characterised by the first year of their availability τ. Following the seminal work of Dixit and Stiglitz (1977), we formulate the aggregate capital stock as:

$$K_t = \left[\int_{t-T}^{t} \left(A_{\tau,t} K_{\tau,t} \right)^{\frac{\varepsilon-1}{\varepsilon}} d\tau \right]^{\frac{\varepsilon}{\varepsilon-1}} \qquad \varepsilon(>1) \tag{2}$$

in which T is the (endogenous) mass of vintages in use, $K_{\tau,t}$ is the amount of capital of vintage τ used in year t (where $t-T \leq \tau \leq t$) and $A_{\tau,t}$ is a vintage-specific productivity parameter. Alternatively, T can be interpreted as the age of the oldest vintage in use. Technological change is embodied in new vintages. The elasticity of substitution between any pair of vintages (in efficiency units) is denoted by ε. Vintages are assumed to be imperfect substitutes ($1 < \varepsilon < \infty$). This formulation implies that firms have an incentive to invest in older technologies, even if new technologies are available that are 'better' when considered in isolation. It is a common characteristic in vintage modelling that it is optimal for firms to invest only in best practice technologies (Meijers 1994).[3] The very reason that old and new vintages coexist in most vintage models is that once firms have incurred the (partly) sunk investment cost, it may not be optimal to replace existing capital goods once a superior technology becomes available. According to equation (2), however, old and new vintages coexist because of a taste for diversity and, hence, firms keep investing in older vintages. The strength of this complementarity effect is defined by the substitution elasticity parameter ε; the larger ε, the lower the degree of complementarity.

The productivity of vintages develops over time according to two factors. The first one is exogenous. As we do not elaborate on the innovation process in the 'intermediate capital' sector but rather focus on the diffusion of the new vintages in the final goods sector we just assume that newer vintages - as they are brought on the market - are more productive than older vintages when those were brought on the market. The second factor is endogenous: vintages improve as they are used. Hence, the productivity endogenously depends on the cumulative investments in vintages. We further label this learning-by-using. More specifically, we assume that the productivity of vintages ($A_{\tau,t}$) develops according to

$$A_{\tau,t} = A_0 e^{g\tau} + \left[1 - (1 + aC_{\tau,t})^{\lambda-1} \right] \left(A_{\tau}^{\max} - A_0 e^{g\tau} \right) \tag{3}$$

[3] A notable exception is Soete and Turner (1984) who model technology diffusion in a macro-economic model.

In this specification, A_0 is initial productivity, g is an exogenously given growth rate of the productivity of new vintages, a measures the strength of learning-by-using effects, C_t represents past cumulative investments in vintage τ ($C_{\tau,t} = \int_{\tau}^{t} K_{\tau,s} ds$), λ represents the curvature of the learning-curve and $A_{\tau}^{\max}$ is the vintage-specific maximum productivity level (that is, the productivity level when the technology has matured). For simplicity, we assume that $A_{\tau}^{\max}$ is in fixed proportion γ (≥1) to the productivity at the date of introduction of the vintage ($A_{\tau}^{\max} = \gamma A_0 e^{g\tau}$). In the special case in which γ=1, the learning-by-using mechanism is absent and productivity of vintages purely depends on the exogenous improvements. The assumption that 0<λ<1 implies that the productivity of a technology in the presence of learning-by-using (γ>1) gradually converges to the mature productivity level $A_{\tau}^{\max}$ once the technology starts to penetrate into the production process. For the time being, we assume for reasons of analytical tractability of the model that learning-by-using is absent (γ=1). We generalise the specification for the productivity development and discuss the implications of learning-by-using for diffusion patterns and adoption of technologies in section 4.2.

As we already mentioned, the use of energy of a particular vintage at a point in time is directly related to the use of capital. Empirical literature supports low substitution possibilities between energy and capital (Kemfert 1998, Kuper and van Soest 1999). For reasons of analytical tractability, we make the extreme assumption of no substitution possibilities:

$$E_{\tau,t} = \psi_{\tau} K_{\tau,t} \qquad \psi_{\tau} > 0 \tag{4}$$

where $E_{\tau,t}$ is the energy input for vintage τ at time t and ψ_{τ} is the vintage-specific energy-capital ratio.[4] The vintage-specific energy-capital ratio is assumed to decline at an exogenously given rate f:

$$\psi_{\tau} = \psi_0 e^{-f\tau} \qquad (f \geq 0) \tag{5}$$

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4 Note that we basically describe the production structure in the final goods sector as a nested Cobb-Douglas function in which capital and energy enter in one nest yielding capital services where substitution possibilities between capital and energy are assumed to be absent (for simplicity).

Newer vintages use less energy than older vintages due to exogenous technological progress. Total energy use in the economy equals $E_t = \int_{t-T}^{t} E_{\tau,t} d\tau$.

Producers in the final goods sector operate under perfect competition and a representative firm maximises profit π :

$$\pi_t = P_{Yt} Y_t - w_t L_{Yt} - \int_{t-T}^{t} p_{K\tau,t} K_{\tau,t} d\tau - \int_{t-T}^{t} p_{Et} E_{\tau,t} d\tau \tag{6}$$

in which P_Y , w, $p_{K\tau}$ and p_E denote the output price, the wage rate, the price of capital goods of a specific vintage and the energy price, respectively (we omit time-indices if possible). In the remainder we assume that the price of energy is exogenously given (determined on the world market). The wage rate is taken to be the numeraire of the model (w=1). Vintage capital is bought from the capital goods sector to which we turn in the next subsection.

2.2 The capital production sector

The capital production sector consists of a mass of T monopolistically competitive firms, each producing a specific vintage according to[5]

$$K_{\tau,t} = L_{K\tau,t} \tag{7}$$

In addition, in each period firms in this sector have to pay a fixed cost in terms of labour (L_f) before being able to produce. Monopoly rents have to compensate for these costs. Firms maximise their profits according to

$$\max \pi_{\tau,t} = p_{K\tau,t} K_{\tau,t} - (L_{K\tau,t} + L_f) w \tag{8}$$

[5] We assume that in each period, a new vintage becomes available due to an exogenous process of technological innovation and only one firm acquires the right to produce capital of this particular vintage. It is of course possible to generalize here and to model a separate sector producing the brands and selling these to the firms producing the capital. In such a setting, firms in the capital production sector would be willing to buy the patent to produce the specific brand and acquire the monopoly right to produce, provided that the profits that can be earned over time are equal to the costs of the patent (compare, for example, Grossman and Helpman, 1991). We think that such a generalization, though interesting, would not add to the basic insights we want to emphasise in this paper.

We assume a constant and exogenous labour supply L. The model is closed by imposing labour market equilibrium:

$$L = L_{Yt} + \int_{t-T}^{t} (L_{K\tau,t} + L_f) d\tau \tag{9}$$

In the next section, we discuss the solution of the model, focusing on the allocation of labour and the determination of the number of vintages used in the production process.

3. Solution of the model

Final goods sector

Producers in the final goods sector perform a standard profit maximisation problem in two stages. First, they determine the optimal relative demand for (the composite of) capital, including energy, and labour by maximising the profit function (6). Using the expression for the use of energy (equation (4)), this results in the standard allocation-rule for a Cobb-Douglas production function implying constant cost shares of capital, including energy costs, and labour:

$$\frac{P_K K_t + p_E \psi K_t}{wL_Y} = \frac{\int_{t-T}^{t} (p_{K\tau} + p_E \psi_\tau) K_\tau d\tau}{wL_Y} = \frac{\alpha}{1-\alpha} \tag{10}$$

in which P_K is the capital index of the composite capital good (we omit time indices when possible). Second, producers decide on the optimal amount of each vintage by solving the following maximisation problem:

$$\max_{K_\tau} \left[\int_{t-T}^{t} \left(A_\tau K_\tau \right)^{\frac{\varepsilon-1}{\varepsilon}} d\tau \right]^{\frac{\varepsilon}{\varepsilon-1}} \quad s.t. \int_{t-T}^{t} \left(p_{K\tau} + p_E \psi_\tau \right) K_\tau d\tau \leq P_K K + p_E E \tag{11}$$

Optimisation yields a downward sloping demand curve for capital of a specific vintage:

$$K_\tau = K_s \left[\frac{A_\tau}{A_s} \right]^{\varepsilon-1} \left[\frac{\left(p_{K\tau} + p_E \psi_\tau \right)}{\left(p_{Ks} + p_E \psi_s \right)} \right]^{-\varepsilon} \tag{12}$$

The relative demand for two vintages of different age (s and τ) thus depends on their relative productivity and their relative prices including energy costs. The 'real' or effective energy costs are determined by the (exogenous) energy price and the energy-capital ratio.

Capital production sector

Producers in the vintage production sector maximise their profits (equation (8)) subject to the downward-sloping demand curve for the vintage that they produce (equation (12)). This results in standard mark-up pricing, according to which the producers of vintages put a mark-up over labour costs and a factor including vintage specific energy costs:

$$\frac{\partial \pi_\tau}{\partial L_{K\tau}} = 0 \Leftrightarrow p_{K\tau} = \frac{\varepsilon}{\varepsilon - 1} w + \frac{p_E \psi_\tau}{\varepsilon - 1} \tag{13}$$

This mark-up is larger the larger the complementarity between different vintages (i.e., the smaller ε) and the larger the vintage specific energy costs of the firms in the final goods sector (i.e. the larger $p_E \psi_\tau$). This basically concludes the description of behaviour of firms in our economy.

Number of vintages and labour allocation

The model is subsequently solved by determining the number of vintages that can be sustained in the economy (that is, the age of the oldest vintage that can be sustained). To understand this intuitively, it is important to notice that newer vintages are more productive than older ones and the producers of vintages have to pay a fixed cost in terms of labour. As a result of the gradual increase in productivity of newer vintages, the (relative) demand for old vintages will gradually decline over time. The complementarity between vintages of different age is the reason that firms do not immediately shift to the most productive vintage. At some point in time, however, the demand for a vintage becomes so low that it can no longer profitably be supplied by the producer of that vintage. Supply will stop and the vintage disappears from the market. This ‘scrapping’ of vintages is - in contrast with the more traditional vintage literature - caused by the impossibilities to profitably *supply* the vintage, whereas the traditional vintage literature explains scrapping from the fact that at some point in time, the vintage can no longer profitably be *used* by the owner. Our model thereby offers an alternative economically motivated and supply oriented explanation for scrapping of vintages that differs from, for example, Den Hartog and Tjan (1980) and Malcolmson (1975).

For determining the number of vintages used, we first need to determine the allocation of labour over the production or assemblage of final goods and the production of vintages, respectively. Using the fact that cost shares of aggregate capital and intermediates (i.e. vintage capital) are constant, we can determine the allocation of labour. For the time being, we assume for reasons of analytical tractability of the model that the energy-capital ratio ψ_τ is constant for all vintages ($\psi_\tau = \psi$). We generalise and discuss the implications of allowing for a vintage-specific energy-capital ratio for diffusion patterns and adoption of technologies in section 4.3.

Using equations (7), (10) and (13), and assuming a constant energy-capital ratio, we derive the allocation of labour as (see Appendix A):

$$L_Y = \frac{(1-\alpha)}{\alpha}\frac{\varepsilon}{\varepsilon-1}\left(1+\frac{p_E\psi}{w}\right)\int_{t-T}^{t} L_{K\tau}d\tau \tag{14}$$

This expression reveals that more assemblage labour will be used relative to labour used for producing vintages, the smaller the share parameter in the production function of final goods, the larger the energy price and the energy-capital ratio, and the lower the elasticity of substitution. The latter is caused by the fact that a low elasticity of substitution results in relatively high prices of vintages due to mark-up pricing and results in a shift from capital to labour in final goods production. An increase in the costs for energy also results in relatively high prices of vintages due to mark-up pricing and results therefore as well in a shift from capital to labour in final goods production. Substituting this equation in the definition for the labour market equilibrium (equation (9)) and rewriting yields the expression for the labour used to produce the vintage capital stock (see Appendix A):

$$\int_{t-T}^{t} L_{K\tau}d\tau = \frac{\alpha(\varepsilon-1)w}{(\varepsilon-\alpha)w+(1-\alpha)\varepsilon p_E\psi}\left(L-\int_{t-T}^{t} L_f d\tau\right) \tag{15}$$

Firms in the capital production sector continue producing their specific vintage as long as they are compensated for their production costs. So they produce as long as

$$p_{K\tau}K_\tau \geq (L_{K\tau}+L_f)w \tag{16}$$

Using the production function for vintages and mark-up pricing (equations (7) and (13)) and assuming a constant energy-capital ratio ($\psi_\tau = \psi$), this expression can be rewritten (with equality) as

$$\frac{\varepsilon}{\varepsilon-1} = \frac{L_{K\tau}+L_f}{L_{K\tau}} - \frac{p_E\psi}{(\varepsilon-1)w} \tag{17}$$

This expression basically determines the minimal required scale of operation for a producer of vintages (and hence, the minimal demand for a particular vintage that is needed for the producer of that vintage to be able to operate profitably). From (17) and using (7) this minimal demand can be derived as

$$\overline{K} = \overline{L} = \left(\frac{\varepsilon - 1}{\frac{p_E \psi}{w} + 1} \right) L_f \tag{18}$$

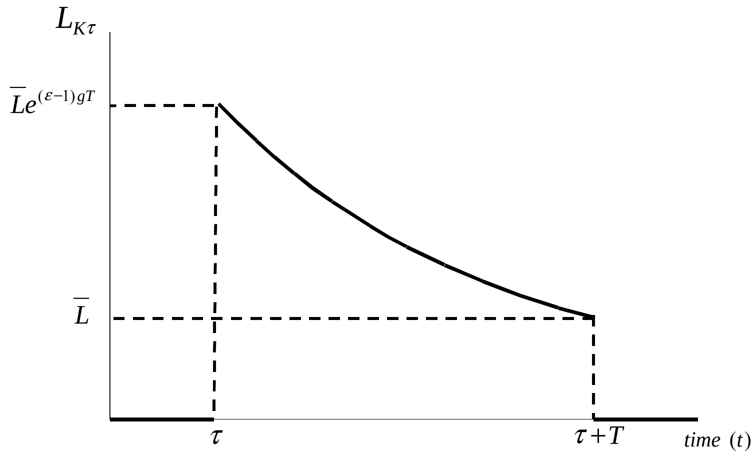
in which $\overline{K} = K_{t-T,t}$ is the amount of the oldest vintage that will be produced and $\overline{L} = L_{Kt-T,t}$ is labour used for its production. Clearly, the minimum scale of operation or the minimal demand for a particular vintage is larger the larger the fixed cost and the larger the elasticity of substitution (and hence the lower the mark-up the producers of the vintages can charge). Any firm that would intend to produce an older vintage for which there would be less demand due to its lower productivity would make losses.

Having determined the production level of the oldest vintage, we can uniquely determine the production levels of more recent vintages which are in use by combining the expression for the relative demand for different vintages and the productivity difference between these vintages. Substituting the expressions for the capital price (equation (13)) into equation (12) and rewriting yields (in the absence of learning-by-using (γ=1) and under the assumption of a constant energy-capital ratio ($\psi_\tau = \psi$)) (see Appendix B):

$$L_{K\tau,t} = \overline{L} e^{g(\varepsilon-1)(\tau+T-t)} \tag{19}$$

Figure 1 graphically illustrates this expression by displaying the production of one particular vintage (arriving on the market at t=τ) over time. This expression reveals that in the presence of exogenous improvements of the performance of newer vintages (g>0), more labour is used for the production of more recent vintages (higher τ). This effect is reinforced when the degree of complementarity among vintages declines (ε increases). We discuss the implications of allowing for a vintage-specific energy-capital ratio ψ_τ (i.e. f>0) for diffusion patterns and adoption of technologies in section 4.3, using a generalised version of (19) (see Appendix B).

Figure 1. Demand for one vintage during its lifetime



The total amount of labour used for the production of vintages with different levels of energy efficiency equals (using equations (18) and (19)):

$$\int_{t-T}^{t} L_{K\tau,t}\,d\tau = \int_{0}^{T} \overline{L}e^{(\varepsilon-1)g\tau}\,d\tau = \frac{\overline{L}\left[e^{(\varepsilon-1)gT}-1\right]}{g(\varepsilon-1)} = \frac{L_f\left[e^{(\varepsilon-1)gT}-1\right]}{g\left(\dfrac{p_E\psi}{w}+1\right)} \tag{20}$$

Combining equations (15) and (20) solves the model for the mass of vintages that can be sustained in the economy (or, alternatively, the age of the oldest vintages in use). This solution for T is given by the following implicit function:

$$\left((\varepsilon-\alpha)w+(1-\alpha)\varepsilon p_E\psi\right)L_f\left[e^{(\varepsilon-1)gT}-1\right]-\alpha g(\varepsilon-1)\left(p_E\psi+w\right)\left[L-TL_f\right]=0 \tag{21}$$

4. Comparative statics

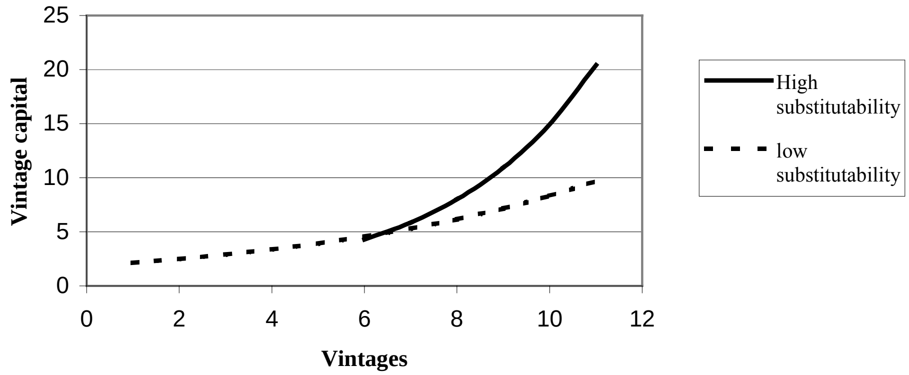
The aim of this section is to illustrate the comparative statics of the model. This will mainly be done by relying on a graphical method that enables us to both illustrate the solution of the model as discussed in the previous section and the comparative statics. In section 4.1, we will discuss the importance of the degree of complementarity between different vintages for understanding the adoption and diffusion of new vintages. In section 4.2, we elaborate on the importance of learning-by-using. This will be done by generalising the development of the productivity of vintages as we introduced it in section 2. In section 4.3, we analyse the adoption and diffusion process of a new energy-efficient technology. More specifically, we analyse how energy-efficient technologies diffuse in the presence of complementarity and how the diffusion process can be affected by energy policies.

4.1 The effects of complementarity

The degree of complementarity is captured by the elasticity of substitution between the vintages. The consequences of an increase in the elasticity of substitution (that is, a lower degree of complementarity) can best be understood by dividing the total effect in three components. (Note that we assume for the time being that there is no learning.) First, increased substitutability reduces the mark-up that producers of intermediates can charge (see equation (13)). Consequently, the minimal demand required for these producers to operate profitably increases. Secondly, increased substitutability implies that the relative demand for vintages is more responsive to increases in productivity of newer vintages (see (12)). Finally, increased substitutability lowers the price of intermediates relative to wages. As a consequence, firms in the final goods sector will, ceteris paribus, increase their demand for intermediates. These three effects can be illustrated in a graph. This is done in Figure 2.[6]

[6] Figure 2 is based on a discretized version of the model with the following parametrization: α=0.5, w=1 (numeraire), g=0.05, A_0=1, γ=0, L=300, L_f=2, ψ=1, p_E=2. The elasticity of substitution is equal to ε=4.2 in the low-complementarity case and ε=7.4 in the high-complementarity case. This results in T=6 and T=11, respectively. Details are available upon request from the authors.

Figure 2. Demand for different vintages in absence of learning



This figure depicts the demand for vintages of different age (on the vertical axis) as a function of the date of introduction on the market (on the horizontal axis). The most recent (current) vintage is located most to the right in the figure. The figure can be understood as follows. Consider first the case in which the elasticity of substitution is low (the dashed line). The upward slope of the line reflects the fact that newer vintages (located more to the right) have a higher productivity and consequently have a higher demand. The demand of the oldest vintage is given as the minimal required demand as defined in equation (17) and is represented by the lowest point on the dashed line. The surface below this line is equal to the amount of labour that is available for the production of vintages as it is given in equation (14). Combining these three elements yields a unique solution of the model which is essentially characterised by the age of the oldest vintage.

Let us now consider what happens when the elasticity of substitution increases. First, the minimal required demand increases because of a reduced mark-up. This implies, ceteris paribus, a reduction of the equilibrium number of vintages. Second, the demand curve gets steeper as the relative demand for vintages becomes more responsive to productivity differences. Ceteris paribus, this implies that the equilibrium number of vintages that can be sustained in the economy declines. Third, producers of final goods shift towards capital as this becomes cheaper. This is reflected by an upward shift of the demand curve. This effect works opposite to the previous two effects and implies that more vintages can be sustained. It can be shown, however, that the former two effects dominate for reasonable parameter values[7], so increased substitutability reduces the number of vintages that can be sustained. The other way around, it can be concluded that complementarity thus slows down the rate of modernisation of the capital stock.

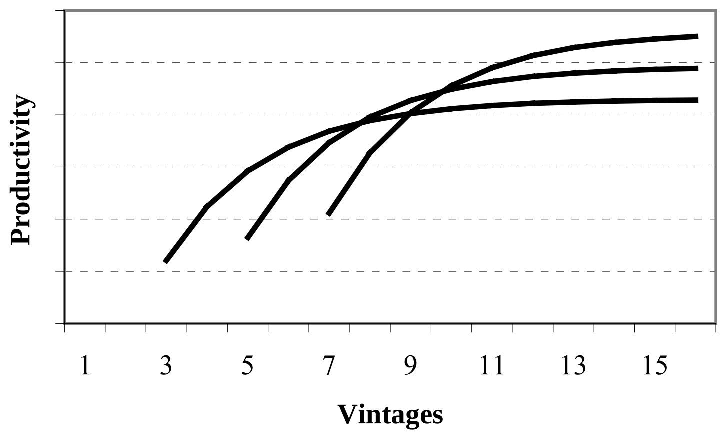
4.2 The effects of learning-by-using

In the previous section, we have assumed a learning effect to be absent for reasons of analytical tractability. In this subsection we include learning-by-using into the model according to the specification given in equation (3). This implies that the productivity of a vintage increases over time as a function of past cumulative investment in that vintage. The productivity improvement is initially relatively large. Furthermore, it is bounded in the sense that there is a vintage-specific maximum productivity level. This formulation captures the available empirical evidence that productivity increases at a relatively fast rate just after the vintage is introduced, in order to slow down at later stages, and levels off when the technology matures (Grübler et al. 1999).

Empirical evidence seems to suggest that the initial learning rate can be quite strong. This implies that situations can arise in which productivity of vintages that have been introduced some periods ago exceeds productivity of vintages that have just been introduced. Figure 3 depicts a typical example of this kind of productivity development of three different vintages. The newer vintage (starting more to the right) is potentially more productive as it comes on the market (at $t=\tau$), but initially the old technology outperforms the new technology due to learning-by-using.

[7] The generality of this result has not yet been proven, but numerical experiments so far have shown that the result is valid for a wide range of parameter values. Details are available from the authors, upon request.

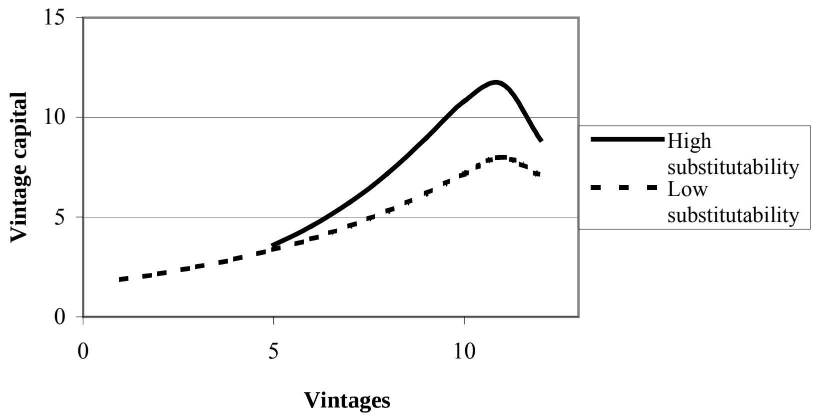
Figure 3. Productivity development of different vintages with learning-by-using



The productivity development reflected in Figure 3 implies that it is possible (dependent on the optimal number of vintages) that there is a vintage of intermediate age that is characterized by the highest productivity. Older vintages are less productive since their learning-by-using potential has declined (or, in other words, those vintages have matured), whereas newer vintages have not yet matured and experienced the productivity improvements due to learning-by-using. The implications of such developments for the diffusion of new technologies are illustrated in figure 4, which is equivalent to Figure 2, but now for the case with learning-by-using.[8]

[8] Figure 4 is based on a discretized version of the model with the following parametrization: α=0.5, w=1 (numeraire), g=0.05, A_0=1, γ=1.25, a=0.2 , λ=0.5, L=300, L_f=2, ψ=1, p_E=2. The elasticity of substitution is equal to ε=3.95 in the low-complementarity case and ε=5.8 in the high-complementarity case. This results in T=12 and T=8 respectively. Details are available upon request from the authors.

Figure 4. Demand for different vintages with learning-by-using



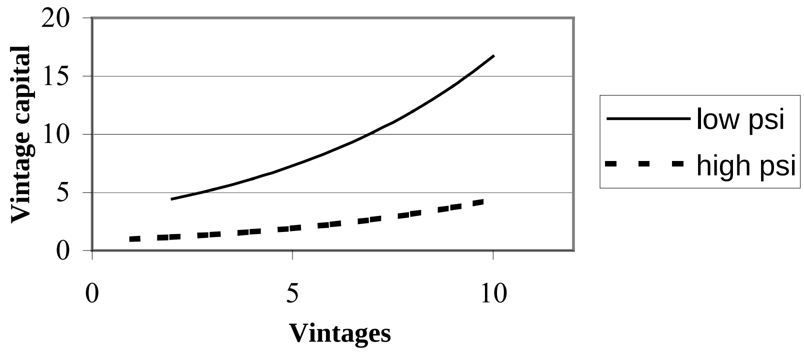
Clearly, new vintages are initially demanded at a relatively limited scale due to their low productivity, but as they improve due to learning-by-using demand will increase, in order to subsequently be gradually phased out of the production process as the vintage matures and ultimately becomes obsolete. The positive effect of learning-by-using on the productivity of older vintages provides a barrier to adopt newer vintages and, hence, slows down the diffusion process. Based upon the similar logic as explained in section 4.1, a higher elasticity of substitution will result in less vintages being used in the production process and at the same time stronger responses to differences in productivity levels between vintages of different age.

4.3 The effects of energy-efficiency and energy price

In this subsection we numerically analyse the effects of changes in energy efficiency and energy price on the diffusion pattern of vintages. First, we compare two economies or two firms that differ from each other in terms of energy efficiency. All other things being equal, we run our model for two constant values of the energy-capital ratio parameter ψ - that is the two economies or firms differ in terms of energy-efficiency, but for each economy or firm the energy-efficiency is constant over all vintages. Figure

5 shows the demand for vintages in absence of learning for a situation with high and low energy-efficiency.[9]

Figure 5. Demand for different vintages under two levels of energy-intensity (in absence of learning)



Because of the fact that energy is complementary to capital (equation (4)) a higher ψ implies that the economy or the firm needs a relatively high amount of energy and is thus relatively energy inefficient. This implies that energy costs comprise a significant part of the costs of the capital-energy composite in the final goods sector. Vice versa this means that the price of vintage capital is of relatively less weight in determining investment decisions of firms. As a result monopolistic producers of vintage capital are able to set a relatively high mark-up (equation (13)): the demand for vintages will react only moderately to an increase in capital price and because of the high mark-up producers of vintages need less demand for their vintage to compensate for their fixed costs (equation (18)). Hence, the economy will sustain a relatively large number of vintages. This is illustrated in Figure 5 with the dashed line. From equation (13) it can be seen that an increase in the energy price, for example because of an energy tax, yields the same result.

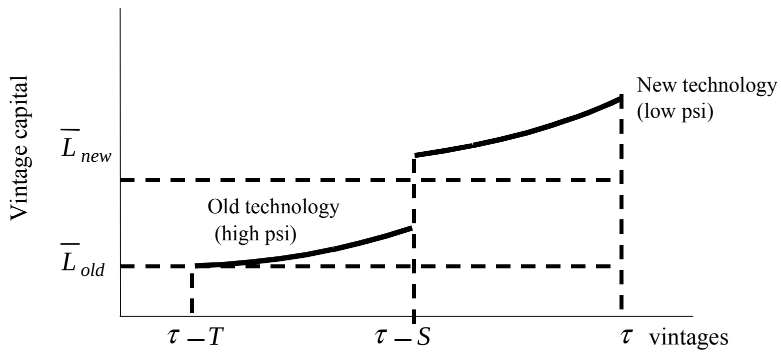
[9] Figure 4 is based on a discretized version of the model with the following parametrization: α=0.5, w=1 (numeraire), g=0.05, A_0=1, γ=0, L=300, L_f=2, ε=4.4, p_E=4. The energy-capital ratio is equal to ψ=0.2 in the low energy-intensitiy case and ψ=4 in the high energy-intensity case. This results in T=9 and T=10, respectively. Details are available upon request from the authors.

The bold line in Figure 5 illustrates the case with relatively high energy efficiency. Here the story is opposite: because of the relative low energy-intensity the price of capital forms a relatively large part of the cost of the capital-energy composite and, hence, the demand for vintages is relatively sensitive to a change in capital price. As a result, the market power of the producers of vintage capital is reduced as compared to the energy-intensive case, which leads to a lower mark-up and thus a reduction in the mass of vintages that is sustained in the economy. Furthermore, a low level of energy-intensity yields a relative low price of intermediates because of the relative low energy costs of the capital-energy composite. As a consequence, firms in the final goods sector employ, ceteris paribus, a relative high demand for vintage capital, leading to a relative high amount of labour used to produce vintage capital (see (14) and (15)). This is reflected by an upward shift of the demand curve for vintages. Applying the same logic, a low energy price yields the same result.

Now we can easily see that imposing an energy tax has two effects. First, it will, ceteris paribus, reduce the amount of energy used because of a lower demand for vintage capital. Second, it will slow down the modernisation of the capital stock in the economy. The first effect results from the increasing costs of the capital-energy composite and the second effect stems from the fact that producers of vintages need less demand to compensate for the fixed costs as they are able to set a higher mark-up. Of course, the first effect depends on the assumption of complementarity between capital and energy and the second effect applies only in case of a constant energy-capital ratio for all vintages.

What happens to a firm when a new energy-saving technology arrives on the market? Of course, the firm will invest in the new technology since the new technology is better, ceteris paribus. Because of the complementarity effect, however, firms continue to invest in old technologies as well and we thus have a transition period. Figure 6 shows a firm's demand for vintages at a particular point in time after the (exogenous) arrival of a new energy-saving technology at time $t=S$.

Figure 6. Demand for vintages during transition to new technology.



T= age of oldest vintage

τ - S = time of introduction of new technology

τ = most recent vintage

The demand curve for vintages essentially combines the two demand curves depicted in Figure 5. The most recent vintages embody the new energy-saving technology while the older vintages embody the older technology. As we argued above, an increase in energy efficiency reduces the mark-up set by the producer of a vintage and, hence, increases the minimal demand $\overline{L}$ needed to compensate for the fixed costs. As a result the demand curve for vintage capital is discontinuous during the transition period.

Finally, we analyse the effect of an increase in energy price on the demand for different vintages when we allow for (exogenous) improvement of energy-efficiency over time. In other words, we relax the assumption of a constant energy-capital ratio, i.e. $f>0$. This means that more recent vintages are less energy-intensive than older vintages. The amount of labour used for the production of vintages can be derived from the generalised version of equation (19) (see Appendix B)

$$L_{K\tau,t} = \overline{L}_t e^{g(\varepsilon-1)(\tau+T-t)} \left[\frac{1+\frac{p_E}{w}\psi_0 e^{-f\tau}}{1+\frac{p_E}{w}\psi_0 e^{-f(t-T)}} \right]^{-\varepsilon} \quad \text{where } T-t<\tau<t,\ e^{-f\tau}<e^{-f(t-T)}$$

This expression reveals that in the presence of exogenous improvements of the energy-efficiency of newer vintages (f>0), more labour is used for the production of more recent vintages (higher τ). This implies that the lines in Figures 5 and 6 get steeper as compared to the case with a constant energy-capital ratio for all vintages (f=0). Let us now consider what happens when the energy price increases, for example as a result of an energy tax. First, the demand for more recent vintages increases, as they are more energy-efficient, which is reflected by a steeper demand curve. Second, producers of final goods shift away from capital as the capital-energy composite becomes more expensive. This is reflected by a downward shift of the demand curve. Third, firms in the final good sector become less sensitive to changes in the capital price because of increased energy costs in the capital-energy composite. This creates increasing market power for the producers of vintages who subsequently set a higher mark-up and, hence, need less demand for their vintage to compensate for their fixed costs. This slows down scrapping of older energy-intensive vintages. This effect works opposite to the previous two effects which speed up scrapping of older vintages. It can be shown, however, that the previous two effects dominate for reasonable parameter values[10], so a higher energy price reduces the number of vintages that can be sustained and above all lowers the demand for vintage capital. As a result, the amount of energy used in the economy is reduced. This conclusion, however, applies only in the short en medium term. Because the capital-energy composite becomes more energy-efficient over time, and thus cheaper, the demand for vintage capital increases over time. Numerical experiments show that for reasonable parameter values the effect of increased energy-efficiency outweighs the effect of a historical increase in the energy price in the long term. As a result, in the long term there is place for extra vintages to be sustained in the economy. Although this leads ceteris paribus to an increase in energy use, the capital stock is relatively energy-efficient in the long term and, hence, the change in energy consumption in the long term depends crucially on the extent of the exogenous energy efficiency improvement (*f*).

5. Conclusion

The widespread adoption of energy-efficient technologies is a lengthy and costly process. In this paper we developed a vintage model to study the diffusion of energy-

[10] Numerical experiments so far have shown that the result is valid for a wide range of parameter values. Details are available from the authors, upon request.

saving technologies and to explain why diffusion is gradual and why firms continue to invest in seemingly inferior technologies. An important characteristic of our model is that vintages are complementary; there are returns to diversity of using different vintages. We have argued that this is a potentially relevant part of the explanation of the energy-efficiency paradox: firms will continue investing in older technologies when newer ones are available. Furthermore, we showed that this effect is intensified when we take a learning-by-using effect into account. A firm faces loss of expertise on a particular vintage, gained by virtue of experience, when it switches to a newer vintage and this provides an extra argument for firms to invest in older vintages.

Our model structure allows for an endogenous determination of the number of vintages a firm uses. In our analysis we show that the stronger the complementarity between different vintages and the stronger the learning-by-using effect, the longer it takes before firms scrap (seemingly) inferior technologies. Furthermore, we show the opposite effect that an increase in energy-efficiency and an increase of the energy price speeds up scrapping of older technologies. Finally, we show that in the presence of continuous energy-efficiency improvements a higher energy price leads to a lower energy consumption in the short and medium term, mainly because of a decrease in the demand for vintage capital. In the long term the energy-efficiency improvement compensates for the higher energy price: the capital-energy composite becomes cheaper which leads to an increasing demand for vintage capital. Since the capital stock is relatively energy-efficient in the long term, an answer to the question as to whether the increase in the capital stock leads to increased energy consumption depends on the extent of the (exogenous) energy efficiency improvement.

Clearly, the simple model developed in this paper could be extended in a number of interesting directions. First, we could allow for the endogenous determination of the rate of learning-by-using, the rate of improvement of new vintages and the increase in energy-efficiency. We refer here to Aghion and Howitt (1996) for such a kind of analysis, drawing a distinction between research (developing new vintages) and development (improving existing vintages). Second, we could allow for the incomplete depreciation of capital in order to assess the importance of complementarity in understanding the development of the stock of capital of different vintages and the investment behaviour of firms. Third, the production structure could be modeled in a more realistic way by allowing for substitution between energy and capital.

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Appendix A

In this appendix we determine the allocation of labour over the production or assemblage of final goods and the production of vintages, respectively. Using the fact that cost shares of capital and intermediates are constant, we can determine the allocation of labour. This results in an expression for the labour used to produce vintage capital. Recall from equation (13) that

$p_{K\tau,t} = \frac{\varepsilon}{\varepsilon-1} w_t + \frac{p_{Et}\psi_\tau}{\varepsilon-1}$ and from equation (7) that $K_{\tau,t} = L_{k\tau,t}$. Substituting (13) and (7) in (10) yields:

$$\frac{\int_{t-T}^{t} \left(\frac{\varepsilon}{\varepsilon-1} w_t + \frac{p_{Et}\psi_\tau}{\varepsilon-1} + p_{Et}\psi_{\tau,t} \quad L_{K\tau,t} d\tau \right.}{w_t L_{Yt}} = \frac{\alpha}{1-\alpha} \tag{A.1}$$

Recall that we assumed that the energy price p_{Et} and the mark-up $\frac{\varepsilon}{\varepsilon-1}$ are uniform for all vintages. Furthermore, recall that we assume for simplicity that the energy-capital ratio ψ_τ is the same for all vintages ($\psi_\tau = \psi$). Subsequently, equation (A.1) reads as:

$$\frac{\left(\frac{\varepsilon}{\varepsilon-1} w_t + \frac{p_{Et}\psi_\tau}{\varepsilon-1} + p_{Et}\psi_{\tau,t} \right) \int_{t-T}^{t} L_{K\tau,t} d\tau}{w_t L_{Yt}} = \frac{\alpha}{1-\alpha} \tag{A.2}$$

Rewriting yields:

$$L_Y = \frac{(1-\alpha)}{\alpha} \frac{\varepsilon}{\varepsilon-1} \left(1 + \frac{p_E\psi_t}{w} \quad \int_{t-T}^{t} L_{K\tau,t} d\tau \right. \tag{A.3}$$

which corresponds to equation (14) in the main text.

Substituting this equation in the definition for the labour market equilibrium (equation (9) in the main text) yields:

$$L = \left[\frac{(1-\alpha)}{\alpha}\frac{\varepsilon}{\varepsilon-1}\left(1+\frac{p_E\psi_t}{w}\right)+1\right]\int_{t-T}^{t} L_{K\tau,t}\,d\tau + \int_{t-T}^{t} L_f\,d\tau \tag{A.4}$$

Rewriting yields the expression for the labour used to produce the capital stock:

$$\int_{t-T}^{t} L_{K\tau,t}\,d\tau = \frac{\alpha(\varepsilon-1)w}{(\varepsilon-\alpha)w+(1-\alpha)\varepsilon p_E\psi_t}\left(L-\int_{t-T}^{t} L_f\,d\tau\right) \tag{A.5}$$

which corresponds to equation (15) in the main text (we omit time indices in the main text).

Appendix B

In this appendix we derive an expression that relates the production of each vintage to the production of the oldest vintage $\bar{K} = K_{t-T,t}$. Substituting equation (13) into equation (12) of the main text yields:

$$K_\tau = K_s\left[\frac{A_\tau}{A_s}\right]^{\varepsilon-1}\left[\frac{w+p_E\psi_\tau}{w+p_E\psi_s}\right]^{-\varepsilon} \tag{B.1}$$

When we take for the vintage with index *s* the oldest vintage with index *t-T* equation (B.1) develops in:

$$L_{K\tau} = K_{t-T}\left[\frac{A_\tau}{A_{t-T}}\right]^{\varepsilon-1}\left(\left[\frac{w+p_E\psi_\tau}{w+p_E\psi_{t-T}}\right]^{-\varepsilon}\right) \tag{B.2}$$

Recall that $K_{\tau,t} = L_{K\tau,t}$ (equation (7) in the main text) and that $\psi_\tau = \psi_0 e^{-f\tau}$ (equation (5) in the main text). Furthermore, recall that we assumed absence of learning, so $A_{\tau,t} = A_0 e^{g\tau}$ (see equation (3)). Substituting these expressions into equation (B.2) yields:

$$L_{K\tau} = L_{t-T}\left[\frac{A_0 e^{g\tau}}{A_0 e^{g(t-T)}}\right]^{\varepsilon-1}\left[\frac{w + P_E \psi_0 e^{-f\tau}}{w + P_E \psi_0 e^{-f(t-T)}}\right]^{-\varepsilon} \tag{B.3}$$

Rewriting yields:

$$L_{K\tau} = L_{t-T} e^{g(\varepsilon-1)(\tau+T-t)}\left[\frac{w + P_{Et} \psi_0 e^{-f\tau}}{w + P_{Et} \psi_0 e^{-f(t-T)}}\right]^{-\varepsilon} \tag{B.4}$$

From equation (18) in the main text it can be seen that the labour used to produce the oldest vintage, $L_{Kt-T,t}$, equals the minimal demand needed for the producer of that vintage to be able to operate profitably, that is $\overline{K} = \overline{L}$. As a result the demand (production) for each vintage can be related to the demand (production) for the oldest vintage in the market according to:

$$L_{K\tau,t} = \overline{L}_t e^{g(\varepsilon-1)(\tau+T-t)}\left[\frac{w + P_E \psi_0 e^{-f\tau}}{w + P_E \psi_0 e^{-f(t-T)}}\right]^{-\varepsilon} \tag{B.5}$$

Rewriting yields:

$$L_{K\tau,t} = \overline{L_t} e^{g(\varepsilon-1)(\tau+T-t)}\left[\frac{1 + \frac{P_E}{w} \psi_0 e^{-f\tau}}{1 + \frac{P_E}{w} \psi_0 e^{-f(t-T)}}\right]^{-\varepsilon} \tag{B.6}$$

where $T - t < \tau < t$, $e^{-f\tau} < e^{-f(t-T)}$. When we assume a constant energy-capital ratio ($\psi_\tau = \psi$), i.e. $f = 0$, the second term at the right hand of equation (B.6) becomes 1 and hence equation (B.6) reduces to

$$L_{K\tau,t} = \overline{L} e^{g(\varepsilon-1)(\tau+T-t)} \tag{B.7}$$

This corresponds to equation (19) in the main text.